Math 381 Complex Variables and Transforms

Instructor: Matthias Nagel due in class 13:35, Mar 29

Homework 5

Exercise 5.1.

1. Compute the inverse Z-transform of the function
$$F(z) := \frac{2}{z-3}.$$
2. Show for a complex number $a \in \mathbb{C}$ that
$$Z[\exp(at)] = \frac{z}{z - \exp(aT)}.$$

Exercise 5.2. Expand the function $f(z) := \frac{1}{(z-1)^3}$ into a power series around $z_0 = 0$.

Exercise 5.3. Calculate the winding numbers of the path γ around the points a_i drawn in Figure 1.

Exercise 5.4.

1. Evaluate the following line integral
$$\int_{\partial B_2(0)} z \sin\left(\frac{1}{z-1}\right) dz.$$
2. Determine the nature of the isolated singularities at z_0 of the functions below. If the function has a pole at z_0 also determine its order.
   a) $\sinh(1/z)$ at $z_0 = 0$.
   b) $\frac{e^z - e}{\log z}$ at $z_0 = 1$, where log denotes the principal branch of the logarithm.
   c) $\frac{\sinh z}{z \sin z}$ at $z_0 = 0$.

Exercise 5.5.

1. Evaluate the line integral
$$\int_{\partial B_5(9)} \frac{1}{\sin\sqrt{z}} dz,$$
where $\sqrt{z}$ denotes the principal branch of the complex square root.
2. Evaluate the integral
$$\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2 + 9} dz.$$
Hint: Use Jordan's Lemma.

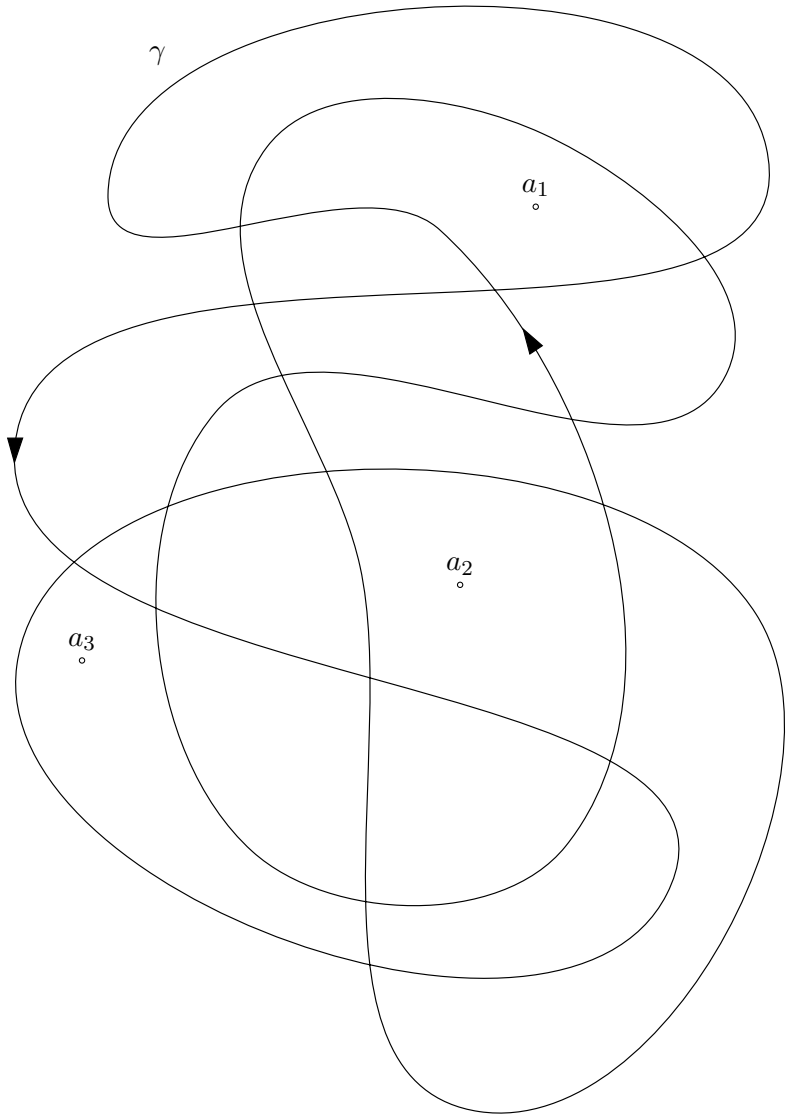

Figure 1: Curve γ and points a_i for Exercise 5.3